A MODEL OF LABOUR MIGRATION AND URBAN UNEMPLOYMENT IN AFRICA

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1. Introduction

Rural to urban labour migration and the high rate of urban unemployment continue to create serious economic and social problems in the developmental processes of many African countries.

The relationships between migration and urban unemployment and the consequences of their interaction on labour mobility are complex and stand in need of clarification and quantitative analysis afforded by mathematical modelling, but few such models have been presented. Todaro (1969) has constructed such a model from the point of view of an economist. It is essentially a deterministic model. Our model, which is thoroughly probabilistic, is more sociological in its orientation, although we take heed of economic considerations. It predicts the pattern of movement of the labour force and can be used to compare the probable consequences of social and economic policies.

2. The Model

Suppose we consider the labour force in an African country as being divided into six states, as follows:

Rural	1.	Unemployed (or underemployed) in the rural sector;
	2.	Employed in agriculture;
	3.	Employed in the rural sector, but not in agriculture;
Urban	4.	Unemployed in the urban sector;
	5.	Employed in the urban sector;
Rural or Urban	6.	Departed from the labour force.

Further refinements can be envisaged, but it would not be fruitful to attempt a finer division than can actually be observed. We assume that it is possible to count the number of people who move from each state to each other state and thereby to estimate the probability of making such a move. This information we summarise in a matrix of transition probabilities, $P=(p_{ij})$, $i,j = 1,\ldots, 6$.

The model as formulated so far is, of course, a finite Markov-chain. But this structure is too coarse. It does not take into account variation of time of stay in each state, which is an important factor in a realistic analysis of migration. Thus, it is a well-known fact of social life that the longer one has been in a particular state the less likely one is to move to another state (the so-called Axiom of Cumulative Inertia) (McGinnis, 1968).

We incorporate these realities into our model by assuming that for every pair of states (i.j) there exists a distribution $F_{ij}(t) = P_r$ going from i to j on or before time t, given that we are going to make that move . For such a process one can calculate numerous quantities of interest, such as the means and the variances of first passage times (the times to go from state i to state j, for example the time for the unemployed country boy to get a job in the city).

If the "embedded Markov-chain" of our process is ergodic, we can calculate the limiting probabilities of each state, the proportion of time which the process spends in each state, in the long run.

For those calculations, we need to know only first and second moments of $F_{ij}(t)$, which we can estimate by collecting data on how many people moved from i to j and how long they waited before doing so.

3. A further assumption on the $F_{ij}(t)$

We have assumed, at least tentatively, that the $F_{ij}(t)$ are gamma distributions, with density

$$f_{\alpha}v(x) = \frac{1}{\Gamma(v)} \alpha^{v} x^{v-1} e^{-\alpha x}, \quad x > 0.$$

For $0 < \alpha < 1$, the gamme distribution is a "decreasing failure rate" distribution, consistent with the "axiom of cumulative inertia." For V=1, it reduces to an exponential distribution. Since some of our transitions will no doubt resemble poisson processes, this generality of the distribution form is appropriate.

We can estimate from a sample the parameters of the $F_{ij}(t)$, evaluate them by numerical integration, and so evaluate the <u>renewal quantities</u>.

$$W(t) = (I - P \times F(t))^{-1} - I$$

(The "x" denotes element-by-element matrix multiplication).

$W_{ij}(t)$ is the mean number of moves into j in $(0,t]$, given that we are in state i at time 0, so that knowledge of $W(t)$ is equivalent to knowledge of how many people are in each state at time t if we know how many people were in each state at time 0.

4. Causal Structure

The semi-Markov model which we have presented is <u>descriptive</u>, although in a quantitative fashion. Given data, it will give us information about what is likely to happen.

We can, as has been suggested by Ginsberg (1972), incorporate causal structure within the semi-Markov framework by expressing some of the parameters of the semi-Markov process as functions of observable "exogenous" variables and seeking to estimate the parameters of these functions, which we shall call "causal functions."

Considering the transition matrix, we can think of the P_{ijs} as being functions of known form of exogenous variables $x_1, \ldots, x_q$ determined by parameters $B_1 \ldots, B_r$ which we must estimate.

We might have, for example,

- x_1 = private investment in the rural sector;
- x_2 = public investment in the rural sector;
- x_3 = private investment in the urban sector;
- x_4 = public investment in the urban sector;
- x_5 = investment in education in rural areas;
- x_6 = investment in education in urban areas.

We shall take the same attitude toward the moments of F (t), or specifically, for reasons which will come out presently, the unconditional mean waiting times.

$$m_i = \Sigma P_{ij}\, m_{ij},$$

where $M = (m_{ij})$ is the matrix of means of F (t).

In our initial investigations we shall assume that the causal functions are of the form

$$P_{ij} = 1 - \exp\left(- \sum_k B_{ijk}\, x_k\right)$$

If we have estimated transition matrices $P^{(\ell)}$, $\ell = 1, \ldots, N$, where the values of ℓ might perhaps correspond to political or geographical divisions, and we have corresponding values of $X^{(\ell)} = (x_1^{(\ell)}, \ldots, x_r^{(\ell)})$ we shall have N equations in r unknowns, an over-determined system which we can solve in the least-squares sense for the B's.

Ginsberg (1972) shows how one can get maximum likelihood estimates of various types of causal functions. Unfortunately, his specific proposals are not directly applicable to our case. He thinks of people making choices of where to move to (or not to move at all). In Nigeria for example, his states might be the nineteen states of the Federation. Ginsberg regards the P_{ij} as increasing functions of the attractiveness of destination j and decreasing functions of a distance, geographical, financial, or social, (d_{ij}). Given our definition of states, the notion of distance is not relevant, and our people have limited choice. Few of them would choose to become unemployed, and fewer still would choose to quit the labour force by illness, death, or going to jail.

5. The Model as an Instrument of Policy

We should like social analysis to be helpful in the rational formulation of social policy. If we can express at least some of the parameters of the semi-Markov process as functions of variables which are accessible to policy, we can move in this direction by using the techniques of Markov-renewal programming.

In this model, or from our point of view, submodel, if we make a move from i to j in time Δt, we receive a reward, positive, negative, or zero, (which in general is conditional on i, j, and Δt). By an iterative scheme of the dynamic programming type, we can find a "policy", i.e. a set of parameters which will maximise either average reward in the long run, if the embedded Markov-chain is ergodic, or expected total reward in transient states, if there are absorbing states.

6. Some Preliminary Results of the Markov-Renewal Programming Model

We have coded the Markov-renewal programming for a computer, in APL, and tried it, using the data shown below. The transition probabilities and mean waiting times in Tables I and II are from a study done by one of our students, Mrs. Ezim Okeke. She considered only five states,

1. Employed in a rural area
2. Unemployed in a rural area
3. Employed in an urban area
4. Unemployed in an urban area
5. Departed from the labour force

She regarded state 5 as being an absorbing state (dead or retired).

The transition probabilities might have been affected by factors such as seasonal changes in employment - occasioning movement away from farming to trading.

Table I: Matrix of Transition Probabilities (P_{ij})

$$P = \begin{bmatrix} 0.79 & 0.01 & 0.10 & 0.05 & 0.05 \\ 0.03 & 0.87 & 0.00 & 0.08 & 0.02 \\ 0.10 & 0.18 & 0.65 & 0.03 & 0.04 \\ 0.15 & 0.10 & 0.16 & 0.55 & 0.04 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Table II: Matrix of Waiting Times

$$M = \begin{bmatrix} 0.90 & 0 & 0.05 & 0.05 & 1.00 \\ 0.10 & 0.65 & 0.07 & 0.18 & 0.85 \\ 0.45 & 0.05 & 0.45 & 0.05 & 0.60 \\ 0.05 & 0.48 & 0.20 & 0.27 & 0.65 \\ 0 & 0 & 0 & 0 & 100.00 \end{bmatrix}$$

The only reason that observational data are needed at all for the Markov-renewal program algorithm is that in practice we want to fix some of the transition probabilities, saying that they represent phenomena which are beyond the reach of public policy.

The matrix displayed in Table III below tells the computer program which probabilities are to be fixed: these are the non-negative entries which, obviously, are identical to the corresponding elements of the transition probability matrix of Table I. The negative numbers in Table IV tell the program where to calculate new probabilities in these positions as elements of an optimal policy.

In this case, in which state 5 is absorbing, the optimal policy is a matrix of transition probabilities and a vector of unconditional mean waiting times which will maximise the total reward earned in passage through transient states. The reward structure is defined by a matrix, an example of which is displayed in Table IV.

Table III: Computed Policy

0.95	0	0	0	0.05	0.855
0.98	0	0	0	0.02	0.098
0.10	0.18	0.68	0	0.04	0.306
0.15	0.10	0.71	0	0.04	0.142
0	0	0	0	1	1

Table IV: Reward Matrix

80	-20	-10	-800	0
30	-20	10	-600	0
20	-10	20	-400	0
160	20	30	-800	0
0	0	0	0	0

The reward matrix expresses our value judgments as to the relative merits of the possible transitions. The matrix of this example says that it is good to stay on the farm or to return to it; bad to move to the city; worse to move to unemployment or to lose one's job, good to get a job. We are indifferent to departure from the labour force.

CONCLUSION

The semi-Markov model enjoys the virtues of flexibility and adaptability. The model does not care what or how many states it has; neither does it care about our choices of the functions by which we relate the parameters of the semi-Markov process to other observable variables or the means by which/estimate the parameters of these functions: we can choose functional forms and estimation procedures as the data and our intuitions about the pertinent social processes direct. Having established such dependencies to our satisfaction, we can use the Markov-renewal programming algorithms to explore the consequences of resource allocation decisions, thus making, at least potentially, practical use of the model.

Human beings are free agents, and their behaviour is stochastic. The semi-Markov model reflects this reality. Within its probabilistic framework we can incorporate modelling of causal structure, expressing some aspects of behaviour as functions of observable, "exogenous" variables while leaving others to direct statistical estimation.

We cannot expect that these techniques will have the neat usefulness that linear programming has to the operation of a refinery; nonetheless, any means by which quantitative estimates of the probable effects of policy may be had should not be scorned. The makers of public policy patently need all the help they can get. A famous example of how a well-meant policy can miscarry is the attempt of the Kenyan Government to reduce urban unemployment problem by subsidising wages, on agreement with employers to increase their labour force. Promptly many, many people rushed to urban areas, and the urban unemployment problem became worse than ever. It is conceivable that the construction of urban housing by the Nigerian Government has had a similar side effect: more shelter available leads to more relatives coming to town.

These plans were made by reasonable men doing their best to choose the least evil, but had they had more means of extrapolation they might have done better.

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